Teorija stacionarne perturbacije

$\hat{H}_0 |\Psi_n^{(0)}\rangle = E_n^{(0)} |\Psi_n^{(0)}\rangle$ - sv. problem se može egzaktno rešiti

$\hat{H} = \hat{H}_0 + \lambda \hat{V}$

$\hat{H} |\Psi_n\rangle = E_n |\Psi_n\rangle$ - nema egzaktnog rešenja (*)

Ideja: Razvijanje u red po malom parametru λ

$|\Psi_n\rangle = |\Psi_n^{(0)}\rangle + \lambda |1\rangle + \lambda^2 |2\rangle + \dots \quad ; \; |\Psi_n^{(0)}\rangle \equiv |n\rangle$

(**)

$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \dots$

$(**) \Rightarrow (*) \Rightarrow$

$$\hat{H}_0 |n\rangle = E_n^{(0)} |n\rangle \quad (1)$$

$$\hat{H}_0 |1\rangle + \hat{V} |n\rangle = E_n^{(0)} |1\rangle + E_n^{(1)} |n\rangle \quad (2)$$

$$\hat{H}_0 |2\rangle + \hat{V} |1\rangle = E_n^{(0)} |2\rangle + E_n^{(1)} |1\rangle + E_n^{(2)} |n\rangle \quad (3)$$

. . . .

Pretpostavka: popravke su ortogonalne na neper-turbovano stanje $\langle n|1\rangle = 0$, $\langle n|2\rangle = 0$, ... odnosno, razvijanjem po bazisu opservable $\hat{H}_0$

$$|1\rangle = \sum_{k \neq n} C_{nk}^{(1)} |k\rangle \quad , \quad |2\rangle = \sum_{k \neq n} C_{nk}^{(2)} |k\rangle$$

U bazisu $|k\rangle$, (2) glasi

$$\hat{H}_0 \sum C_{nk}^{(1)} |k\rangle + \hat{V}|n\rangle = E_n^{(0)} \sum_k C_{nk}^{(1)} |k\rangle + E_n^{(1)} |n\rangle \quad (4)$$

$$\Downarrow$$

$$\sum_k \left(E_k^{(0)} - E_n^{(0)}\right) C_{nk}^{(1)} |k\rangle + \hat{V}|n\rangle = E_n^{(1)} |n\rangle \quad (5)$$

Projekcija (4) na $\langle n|$ daje

$$E_n^{(1)} = \langle n|\hat{V}|n\rangle$$

Ostale projekcije na $\langle m| \neq \langle n|$ daju iz (5)

$$\left(E_m^{(0)} - E_n^{(0)}\right) C_{nm}^{(1)} + V_{mn} = 0$$

$$C_{nm}^{(1)} = \frac{\langle m|\hat{V}|n\rangle}{E_n^{(0)} - E_m^{(0)}} \quad \text{— koeficijent} \quad (*)$$

pa je

$$|1\rangle = \sum_{k\neq n} \frac{\langle k|\hat{V}|n\rangle}{E_n^{(0)} - E_k^{(0)}} |k\rangle \quad (7)^{(*)\text{pozadi}}$$

Iz $(*)$ $\Rightarrow$ uslov primenljivosti teorije perturbacija popravke talasne f-je su male ako je $C_{nm}^{(1)} \ll 1$, odnosno

$$\langle m|\hat{V}|n\rangle \ll E_n^{(0)} - E_m^{(0)}$$

Iz (3), projektovanjem na $\langle n|$ sledi

$$E_n^{(2)} = \langle n|\hat{V}|1\rangle \overset{(5)}{=} \sum_{k\neq n} \frac{|\langle k|\hat{V}|n\rangle|^2}{E_n^{(0)} - E_k^{(0)}} \quad (8)$$

Popravka za energiju prvog reda za energiju degenerisanog energijskog nivoa

Po analogiji sa (2)

$$\hat{H}_0|1\rangle + \hat{V}|0\rangle = E_n^{(0)}|1\rangle + E_n^{(1)}|0\rangle \qquad (8)$$

$|0\rangle$ pripada potprostoru. Uvedimo projektor $\hat{P}_0$, koji projektuje na dati potprostor i delujmo na (8)

$$\hat{P}_0\hat{H}_0|1\rangle + \hat{P}_0\hat{V}|0\rangle = \cancelto{0}{E_n^{(0)}\hat{P}_0|1\rangle} + E_n^{(1)}\hat{P}_0|0\rangle$$

$$\hat{P}_0|0\rangle = |0\rangle, \quad [\hat{P}_0, \hat{H}_0] = 0$$

$$\cancelto{0}{\hat{H}_0\hat{P}_0|1\rangle} + \underbrace{\hat{P}_0\hat{V}\hat{P}_0}_{\hat{V}'}|0\rangle = E_n^{(1)}\hat{P}_0|0\rangle$$

$$\hat{V}'|0\rangle = E_n^{(1)}|0\rangle$$

Popravke su svojstvene vrednosti redukovanog operatora potencijala

Primedba: $\langle\psi_n|\psi_n\rangle = 1$, $|\psi_n\rangle = |n\rangle + \lambda|1\rangle$

$$\left.\begin{aligned} \langle\psi_n|\psi_n\rangle &= 1 + \lambda\langle 1|n\rangle + \lambda\langle n|1\rangle + \cancelto{0}{\lambda^2\langle 1|1\rangle} \\ \langle\psi_n|\psi_n\rangle &= 1 \end{aligned}\right\} \Rightarrow \underline{\langle 1|n\rangle = 0}$$

1. Za LHO je data perturbacija

$$\hat{V} = a\hat{x}^2 + b\,\hat{p}_x$$

Naći prvu popravku za energiju n-tog stanja.

$$E^{(1)} = \langle n|\hat{V}|n\rangle$$

$$= \langle n|\,a\hat{x}^2 + b\,\hat{p}_x\,|n\rangle = a\langle n|\hat{x}^2|n\rangle + b\langle n|\hat{p}_x|n\rangle$$

Od ranije (VI) $\langle n|\hat{x}^2|n\rangle = \dfrac{2n+1}{2\alpha}$ i

$\langle n|\hat{p}_x|n\rangle = 0$

$$E^{(1)} = a\,\frac{2n+1}{2\alpha}$$

Prema tome, $\hat{H} = \dfrac{\hat{p}_x^2}{2m} + \dfrac{1}{2}m\omega^2\hat{x}^2 + a\hat{x}^2 + b\,\hat{p}_x$

a svojstvena energija n-tog nivoa u prvoj aproksimaciji je

$$E_n = \left(n+\tfrac{1}{2}\right)\hbar\omega + \frac{a}{2\alpha}(2n+1)\ ;\ \alpha = \frac{m\omega}{\hbar}$$

2. Dat je DLHO i perturbacije

a) $\hat{x}\hat{y}$

b) $a\hat{x}^2 + b\hat{y}^3$

c) $\alpha\hat{x}^2\hat{y} + \beta\hat{x}\hat{y}^3$ $\Big\}$ Domaći

Naći prve popravke za energiju osnovnog stanja.

---

a) $\hat{V} = \hat{x}\hat{y}$

$n=0$, $n_x=0 + n_y=0 = 0$

Stanje $|n_x=0, n_y=0\rangle$

Popravka

$$E^{(1)} = \langle 0,0|\hat{V}|0,0\rangle$$

$$= \int_{-\infty}^{+\infty} \psi_0(\xi)\, x\, \psi_0(\xi)\, d\xi \int_{-\infty}^{+\infty} \psi_0(\eta)\, y\, \psi_0(\eta)\, d\eta$$

$\Big[\because \xi = \sqrt{\alpha}\, x , \quad \eta = \sqrt{\alpha}\, y \Big]$

$$= \frac{1}{\alpha} \int_{-\infty}^{+\infty} \psi_0(\xi)\, \xi\, \psi_0(\xi)\, d\xi \int_{-\infty}^{+\infty} \psi_0(\eta)\, \eta\, \psi_0(\eta)\, d\eta$$

$$= \frac{1}{\alpha} \int_{-\infty}^{+\infty} \psi_0(\xi) \left(0 + \sqrt{\tfrac{1}{2}}\, \psi_1(\xi)\right) d\xi \int_{-\infty}^{+\infty} \psi_0(\eta) \left(0 + \sqrt{\tfrac{1}{2}}\, \psi_1(\eta)\right) d\eta$$

$$= \frac{1}{2\alpha} \int_{-\infty}^{+\infty} \psi_0(\xi)\psi_1(\xi)\, d\xi \int_{-\infty}^{+\infty} \psi_0(\eta)\, \psi_1(\eta)\, d\eta = 0$$

3. Dat je DIHO. Za perturbaciju iz prethodnog zadatka izračunati popravke I reda za energiju za prvo pobuđeno stanje.

(izvrsti zadatak do kraja)

Prvo pobuđeno stanje $n=1$

$n_x = 0 + n_y = 1 = 1$

$n_x = 1 + n_y = 0 = 1$

Stanja

$|n_x = 0\rangle |n_y = 1\rangle = |1\rangle$

$|n_x = 1\rangle |n_y = 0\rangle = |2\rangle$

Sekularna jednačina

$$\begin{vmatrix} \langle 1|\hat{V}|1\rangle - E^{(1)} & \langle 1|\hat{V}|2\rangle \\ \langle 2|\hat{V}|1\rangle & \langle 2|\hat{V}|2\rangle - E^{(1)} \end{vmatrix} = 0$$

$$\hat{V} = a\hat{x}^2 + b\hat{y}^3$$

Matrični elementi

$$\langle 1|\hat{V}|1\rangle = a\langle 1|\hat{x}^2|1\rangle + b\langle 1|\hat{y}^3|1\rangle$$

$$= a\langle n_x=0|\hat{x}^2|n_x=0\rangle + b\langle n_y=1|\hat{y}^3|n_y=1\rangle$$

$$= \frac{a}{\alpha}\int_{-\infty}^{+\infty}\psi_0(\xi)\,\xi^2\psi_0(\xi)\,d\xi + \frac{b}{\alpha^{3/2}}\int_{-\infty}^{+\infty}\psi_1(\eta)\,\eta^3\psi_1(\eta)\,d\eta$$

$$\langle 1|\hat{V}|2\rangle = a\langle 1|\hat{x}^2|2\rangle + b\langle 1|\hat{y}^3|2\rangle$$

$$= \frac{a}{\alpha}\int_{-\infty}^{+\infty}\psi_0(\xi)\,\xi^2\psi_1(\xi)\,d\xi\int_{-\infty}^{+\infty}\psi_0(\eta)\,\psi_1(\eta)\,d\eta$$

$$+ \frac{b}{\alpha^{3/2}}\int_{-\infty}^{+\infty}\psi_0(\eta)\,\eta^3\psi_1(\eta)\,d\eta\int_{-\infty}^{+\infty}\psi_0(\xi)\,\psi_1(\xi)\,d\eta = 0$$

Slično kao gore

$$\langle 2|\hat{V}|1\rangle = 0$$

$$\langle 2|\hat{V}|2\rangle = a\langle 2|\hat{x}^2|2\rangle + b\langle 2|\hat{y}^3|2\rangle$$

$$= a\langle n_x=1|\hat{x}^2|n_x=1\rangle + b\langle n_y=0|\hat{y}^3|n_y=0\rangle$$

$$= \frac{a}{\alpha}\int_{-\infty}^{+\infty}\psi_1(\xi)\,\xi^2\psi_1(\xi)\,d\xi + \frac{b}{\alpha^{3/2}}\int_{-\infty}^{+\infty}\psi_0(\eta)\,\eta^3\psi_0(\eta)\,d\eta$$

Nađene matrične elemente zamijeniti u gornju determinantu i izračunati popravke.

4. Za LHO je data perturbacija

$$\hat{V} = a\hat{x}^2 + b\hat{p}_x$$

Naći prvu popravku za stanje osnovnog nivoa a onda i drugu popravku za energiju osnovnog nivoa

Prva popravka za stanje

$$|1\rangle = \sum_{m\neq n}\sum_{\lambda_m} \frac{\langle \Psi_{m\lambda_m}^{(0)} | \hat{V} | \Psi_n^{(0)} \rangle}{E_n^{(0)} - E_m^{(0)}} | \Psi_{m\lambda_m}^{(0)} \rangle$$

Za LHO su svojstvene vrednosti nedegenerisane i nema sumiranja po λ_m

$$|1\rangle = \sum_{m\neq n} \frac{\langle \Psi_m^{(0)} | \hat{V} | \Psi_n^{(0)} \rangle}{E_n^{(0)} - E_m^{(0)}} | \Psi_m^{(0)} \rangle$$

Matrični elementi

$$\langle \Psi_m^{(0)} | \hat{V} | \Psi_n^{(0)} \rangle =$$

$$a\langle \Psi_m^{(0)} | \hat{x}^2 | \Psi_n^{(0)} \rangle + b\langle \Psi_m^{(0)} | \hat{p}_x | \Psi_n^{(0)} \rangle =$$

i po zadatku $n=0$!

$$= a\langle \Psi_m^{(0)} | \hat{x}^2 | \Psi_0^{(0)} \rangle + b\langle \Psi_m^{(0)} | \hat{p}_x | \Psi_0^{(0)} \rangle$$

$$\langle \psi_m^{(0)} | \hat{x}^2 | \psi_0^{(0)} \rangle = \frac{1}{\alpha} \int_{-\infty}^{+\infty} \psi_m^{(0)}(\xi)\, \xi^2 \psi_0^{(0)}(\xi)\, d\xi$$

$$= \frac{1}{\alpha} \int_{-\infty}^{+\infty} \psi_m^{(0)}(\xi)\xi\left(0 + \sqrt{\frac{1}{2}}\, \psi_1^{(0)}(\xi)\right) d\xi$$

$$= \frac{1}{\sqrt{2}\alpha} \int_{-\infty}^{+\infty} \psi_m^{(0)}(\xi)\, \xi \psi_1^{(0)}(\xi)\, d\xi =$$

$$= \frac{1}{\sqrt{2}\alpha} \int_{-\infty}^{+\infty} \psi_m^{(0)}(\xi) \left(\sqrt{\frac{1}{2}}\, \psi_0^{(0)}(\xi) + \psi_2^{(0)}(\xi) \right) d\xi$$

$$= \frac{1}{\sqrt{2}\alpha} \left(\sqrt{\frac{1}{2}}\, \delta_{m0} + \delta_{m2} \right)$$

$$\langle \psi_m^{(0)} | \hat{P}_x | \psi_0^{(0)} \rangle = \sqrt{\alpha}(-i\hbar) \int_{-\infty}^{+\infty} \psi_m^{(0)}(\xi)\, \frac{d}{d\xi}\psi_0(\xi)\, d\xi$$

$$= -i\hbar\sqrt{\alpha} \int_{-\infty}^{+\infty} \psi_m^{(0)}(\xi) \left(0 - \sqrt{\frac{1}{2}}\, \psi_1^{(0)}(\xi) \right) d\xi$$

$$= i\hbar\sqrt{\frac{\alpha}{2}} \int_{-\infty}^{+\infty} \psi_m^{(0)}(\xi)\, \psi_1^{(0)}(\xi)\, d\xi$$

$$= i\hbar\sqrt{\frac{\alpha}{2}}\, \delta_{m1}$$

$$|1\rangle = \sum_{m \neq 0} \frac{1}{E_0^{(0)} - E_m^{(0)}} \left(\frac{a}{\sqrt{2}\alpha} \left(\sqrt{\frac{1}{2}}\, \delta_{m0} + \delta_{m2} \right) + bi\hbar \sqrt{\frac{\alpha}{2}}\, \delta_{m1} \right) |\psi_m^{(0)}\rangle$$

$$|1\rangle = \sum_{m\neq 0} \frac{1}{E_0^{(0)} - E_m^{(0)}} \frac{a}{2\alpha} \delta_{m0}^{\;0} |\psi_m^{(0)}\rangle +$$

$$\sum_{m\neq 0} \frac{1}{E_0^{(0)} - E_m^{(0)}} \frac{a}{\sqrt{2}\alpha} \delta_{m2} |\psi_m^{(0)}\rangle +$$

$$\sum_{m\neq 0} \frac{b i\hbar}{E_0^{(0)} - E_m^{(0)}} \delta_{m1} \sqrt{\frac{\alpha}{2}} |\psi_m^{(0)}\rangle \Rightarrow$$

$$|1\rangle = \frac{1}{E_0^{(0)} - E_2^{(0)}} \frac{a}{\sqrt{2}\alpha} |\psi_2^{(0)}\rangle +$$

$$\frac{1}{E_0^{(0)} - E_1^{(0)}} i\hbar b \sqrt{\frac{\alpha}{2}} |\psi_1^{(0)}\rangle$$

$$E_n^{(0)} = \left(n + \frac{1}{2}\right)\hbar\omega$$

$$\left.\begin{aligned} E_0^{(0)} &= \frac{1}{2}\hbar\omega \\ E_1^{(0)} &= \frac{3}{2}\hbar\omega \\ E_2^{(0)} &= \frac{5}{2}\hbar\omega \end{aligned}\right\} \Rightarrow \begin{aligned} E_0^{(0)} - E_2^{(0)} &= -2\hbar\omega \\ E_0^{(0)} - E_1^{(0)} &= -\hbar\omega \end{aligned}$$

$$|1\rangle = \frac{1}{-2\hbar\omega} \frac{a}{\sqrt{2}\sqrt{\frac{m\omega}{\hbar}}} |\psi_2^{(0)}\rangle +$$

$$\frac{1}{-\hbar\omega} \frac{i\hbar b}{\sqrt{2}} \left(\frac{m\omega}{\hbar}\right)^{\frac{1}{4}} |\psi_1(0)\rangle$$

$$E_n^{(2)} = \langle n | \hat{V} | 1 \rangle \quad ; \quad |n\rangle \equiv |\Psi_n^{(0)}\rangle$$

$$|1\rangle = c_1 \, |\Psi_1^{(0)}\rangle + c_2 \, |\Psi_2^{(0)}\rangle$$

$$E_0^{(2)} = \langle \Psi_0^{(0)} | \hat{V} | \left(c_1 \, |\Psi_1^{(0)}\rangle + c_2 \, |\Psi_2^{(0)}\rangle \right)$$

$$= c_1 \langle \Psi_0^{(0)} | \hat{V} | \Psi_1^{(0)} \rangle + c_2 \langle \Psi_0^{(0)} | \hat{V} | \Psi_2^{(0)} \rangle$$

$$= c_1 \langle \Psi_0^{(0)} | a\hat{x}^2 + b\,\hat{p}_x | \Psi_1^{(0)} \rangle + c_2 \langle \Psi_0^{(0)} | a\hat{x}^2 + b\hat{p}_x | \Psi_2^{(0)}$$

$$= a c_1 \langle \Psi_0^{(0)} | \hat{x}^2 | \Psi_1^{(0)} \rangle + b c_1 \langle \Psi_0^{(0)} | \hat{p}_x | \Psi_1^{(0)} \rangle$$
$$+ a c_2 \langle \Psi_0^{(0)} | \hat{x}^2 | \Psi_2^{(0)} \rangle + b c_2 \langle \Psi_0^{(0)} | \hat{p}_x | \Psi_2^{(0)} \rangle$$

itd.

5 Za TIHO zadate je perturbacija

$$\hat{V} = \hat{x}\hat{p}_y\hat{z}^2$$

Naći popravku 1. reda za energiju osnovnog stanja.

$$E^{(1)} = \langle 0 | \hat{V} | 0 \rangle$$

$$|0\rangle \rightarrow \psi_0(\xi) \quad ; \; n = 0 \, , \quad n = n_x + n_y + n_z$$

$$|0\rangle \equiv |n_x = 0\rangle |n_y = 0\rangle |n_z = 0\rangle$$

$$E^{(1)} = \langle n_x = 0 | \hat{x} | n_x = 0 \rangle \langle n_y = 0 | \hat{p}_y | n_y = 0 \rangle \langle n_z = 0 | \hat{z}^2 | n_z = 0 \rangle$$

$$\langle n_x = 0 | \hat{x} | n_x = 0 \rangle = \int_{-\infty}^{+\infty} \psi_0(\xi) \frac{\xi}{\sqrt{\alpha}} \psi_0(\xi) \, d\xi$$

$$\langle n_y = 0 | \hat{p}_y | n_y = 0 \rangle = \int_{-\infty}^{+\infty} \psi_0(\eta) \sqrt{\alpha} \frac{d\psi_0(\eta)}{d\eta} \, d\eta$$

$$\langle n_z = 0 | \hat{z}^2 | n_z = 0 \rangle = \int_{-\infty}^{+\infty} \psi_0(\zeta) \frac{1}{\alpha} \zeta^2 \psi_0(\zeta) \, d\zeta$$

Domaći

ζ - zeta ;

ξ - ksi

η - eta

6. Za TIHO je zadata perturbacija

$$\hat{V} = \hat{x}\hat{p}_y\hat{z}$$

Naći popravku prvog reda za energiju za prvo pobuđeno stanje.

$n = 1 \Rightarrow$

$n_x = 1 + n_y = 0 + n_z = 0 = 1$

$n_x = 0 + n_y = 1 + n_z = 0 = 1$

$n_x = 0 + n_y = 0 + n_z = 1 = 1$

Svojstvena vrednost E_1 je degenerisana.

Stanja:

$|1\rangle = |1\,0\,0\rangle$

$|2\rangle = |0\,1\,0\rangle$

$|3\rangle = |0\,0\,1\rangle$

Sekularna jna

$$\begin{vmatrix} \langle 1|\hat{V}|1\rangle - E^{(1)} & \langle 1|\hat{V}|2\rangle & \langle 1|\hat{V}|3\rangle \\ \langle 2|\hat{V}|1\rangle & \langle 2|\hat{V}|2\rangle - E^{(1)} & \langle 2|\hat{V}|3\rangle \\ \langle 3|\hat{V}|1\rangle & \langle 3|\hat{V}|2\rangle & \langle 3|\hat{V}|3\rangle - E^{(1)} \end{vmatrix} = 0$$

$$\langle 1|\hat{V}|1\rangle = \langle 1|\hat{x}|1\rangle \langle 0|\hat{p}_y|0\rangle \langle 0|\hat{z}|0\rangle$$

$$\langle 1|\hat{V}|2\rangle = \langle 1|\hat{x}|0\rangle \langle 0|\hat{p}_y|1\rangle \langle 0|\hat{z}|0\rangle$$

$$\langle 1|\hat{V}|3\rangle = \langle 1|\hat{x}|0\rangle \langle 0|\hat{p}_y|0\rangle \langle 0|\hat{z}|1\rangle$$

Slično oromE i preostale dve vrste.
Tri rešenja za $E^{(1)}$ se nalaze rešavanjem determinante.

Za pismeni deo ispita

Ista postavka, ~~po~~ ali druga popravka energije za osnovno stanje.
To bi značilo da se nađe $|1\rangle$

$$E^{(2)} = \langle \psi_n^{(0)} | \hat{V} | 1 \rangle$$

pre toga?

Domaći:
Zadata je perturbacija

$$\hat{V} = \alpha \hat{p}_x + \beta \hat{y}^2 + \gamma \hat{z}$$

za TIHO. Naći 1. popravku za energiju osnovnog stanja

Izračunati 1. popravku energije prvog pobuđenog ~~nivoa~~ stanja vodonika u slabom spoljašnjem elektrostatičkom polju (linearni Štarkov efekat) duž z-ose.

Spinski stepeni slobode nisu od interesa, jer interaguju samo sa magnetnim poljem.

(Npr. Stern-Gerlah)

Prvo pobuđeno stanje : $n=2 \Rightarrow$

$|200\rangle = |1\rangle$

$|210\rangle = |2\rangle$

$|211\rangle = |3\rangle$

$|21{-1}\rangle = |4\rangle$

Perturbacija

$$\hat{V} = -\underline{\vec{p}}\cdot\vec{E} = -e\hat{\vec{r}}\cdot\vec{E} = -eE_z\hat{z}$$

(Električno polje duž z ose)

Sekularna j-na

$$\begin{vmatrix} \langle 1|\hat{V}|1\rangle - E^{(1)} & \langle 1|\hat{V}|2\rangle & \langle 1|\hat{V}|3\rangle & \langle 1|\hat{V}|4\rangle \\ \langle 2|\hat{V}|1\rangle & \langle 2|\hat{V}|2\rangle - E^{(1)} & \langle 2|\hat{V}|3\rangle & \langle 2|\hat{V}|4\rangle \\ \langle 3|\hat{V}|1\rangle & \langle 3|\hat{V}|2\rangle & \langle 3|\hat{V}|3\rangle - E^{(1)} & \langle 3|\hat{V}|4\rangle \\ \langle 4|\hat{V}|1\rangle & \langle 4|\hat{V}|2\rangle & \langle 4|\hat{V}|3\rangle & \langle 4|\hat{V}|4\rangle - E^{(1)} \end{vmatrix} =$$

$$\langle 1|\hat{V}|1\rangle = -eE_z\langle 200|\hat{z}|200\rangle$$

$$\langle 200 | \hat{Z} | 200 \rangle = \int_0^\infty \int_0^\pi \int_0^{2\pi} R_{20}^*(r) Y_0^{0*}(\theta,\varphi) \, (r\cos\theta)$$

$$R_{20}(r) Y_{10}^{0}(\theta,\varphi) \, r^2 \sin\theta \, dr \, d\theta \, d\varphi =$$

$$= \int_0^\infty R_{20}^*(r) R_{20}(r) r^2 dr \cdot \int_0^\pi \int_0^{2\pi} Y_{10}^{0}(\theta,\varphi) \cos\theta \, Y_{10}^{0}(\theta,\varphi)$$

$$\sin\theta \, d\theta \, d\varphi$$

$$\left[\cos\theta \, Y_l^m = \sqrt{\frac{(l+|m|+1)(l-|m|+1)}{(2l+1)(2l+3)}} \, Y_{l+1}^m + \right.$$

$$\left. \sqrt{\frac{(l-|m|)(l+|m|)}{(2l-1)(2l+1)}} \, Y_{l-1}^m \right]$$

$$= \int_0^{+\infty} R_{20}^*(r) R_{20}(r) r^2 dr \int_0^\pi \int_0^{2\pi} Y_{10}^{0*}(\theta,\varphi) \left[\sqrt{\frac{1\cdot 1}{1\cdot 3}} \, Y_1^0 + \right.$$

$$\left. \sqrt{\frac{0\cdot 0}{-1\cdot 1}} \, Y_{-1}^0 \right] \sin\theta \, d\theta \, d\varphi =$$

$$= \int_0^{+\infty} R_{20}^*(r) R_{20}(r) r^2 dr \int_0^\pi \int_0^{2\pi} Y_{10}^{0*}(\theta,\varphi) \frac{1}{\sqrt{3}} Y_1^0(\theta,\varphi) \sin\theta \, d\theta \, d\varphi =$$

$$= \dots$$

Za domaći

Primedba: U nerelativističkoj aproksimaciji, električno polje ne interaguje sa spinom. Ova aproksimacija je važana za električna polja slabija od 10^3 V/cm.

Naći popravku 1. reda za energiju osnovnog stanja vodonika u slabom spolj. mag. polju (Spinski Zemanov) duž z-ose.

$n=1 \Rightarrow |100\rangle$

Sa spinom, imamo stanja $|100\tfrac{1}{2}\rangle = |1\rangle$
$|100-\tfrac{1}{2}\rangle = |2\rangle$

$$\hat{V} = -\mu_B (g_e \hat{\vec{L}} + g_s \hat{\vec{S}})\vec{B}$$

$g_e = 1, \quad g_s = 2$

Neka je magnetno polje $\vec{B} = (0,0,B_z)$

$$\hat{V} = -\mu_B (\hat{L}_z B_z + 2\hat{S}_z B_z)$$

Sekularna j-na

$$\begin{vmatrix} V_{11} - E^{(1)} & V_{12} \\ V_{21} & V_{22} - E^{(1)} \end{vmatrix} = 0$$

$$V_{11} = \langle 1|\hat{V}|1\rangle = \langle 100\tfrac{1}{2}|\hat{V}|100\tfrac{1}{2}\rangle$$

$$= \langle 100\tfrac{1}{2}|-\mu_B(\hat{L}_z B_z + 2\hat{S}_z B_z)|100\tfrac{1}{2}\rangle$$

$$= -\mu_B B_z \langle 100\tfrac{1}{2}|\hat{L}_z|100\tfrac{1}{2}\rangle - 2\mu_B B_z \langle 100\tfrac{1}{2}|\hat{S}_z|100\tfrac{1}{2}\rangle$$

$$= -\mu_B \cdot 0 - 2\mu_B B_z \frac{\hbar}{2} = -\mu_B B_z \hbar$$

Slicno,

$$V_{22} = \mu_B B_z \hbar$$

Vandijagonalni elementi su nula (Proveriti)

$$\begin{vmatrix} -\mu_B B_z \hbar - E^{(1)} & 0 \\ 0 & \mu_B B_z \hbar - E^{(1)} \end{vmatrix} = 0$$

$$E_1^{(1)} = -\mu_B B_z \hbar \qquad E_2^{(1)} = +\mu_B B_z \hbar$$

1. Naći prvu popravku energije za prvo pobuđeno stanje vodonika u slabom spoljašnjem magnetnom polju, duž z-ose. Spin ne uzimati u obzir (Orbitalni Zeemanov efekat)

Prvo pobuđeno stanje vodonika $n=2$

$n=2, \quad l=0,1 \quad , \quad (m_l=0), \quad (m_l=-1,0,1)$

$$\left.\begin{aligned} |1\rangle &= |200\rangle \\ |2\rangle &= |210\rangle \\ |3\rangle &= |211\rangle \\ |4\rangle &= |21{-1}\rangle \end{aligned}\right\} E_2 \text{ (degenerisana svoj. vrednost)}$$

$$\hat{V} = -\mu_B g_l \hat{\vec{L}} \vec{B} \quad , \quad g_l = 1$$

Neka je magnetno polje $\vec{B} = (0,0,B_z)$

$$\hat{V} = -\mu_B \hat{L}_z B_z$$

Sekularna j-na

$$\begin{vmatrix} V_{11}-E^{(1)} & V_{12} & V_{13} & V_{14} \\ V_{21} & V_{22}-E^{(1)} & V_{23} & V_{24} \\ V_{31} & V_{32} & V_{33}-E^{(1)} & V_{34} \\ V_{41} & V_{42} & V_{43} & V_{44}-E^{(1)} \end{vmatrix} = 0$$

$$V_{ij} = \langle i | \hat{V} | j \rangle \quad , \quad i = \overline{1,n} \; , \; j = \overline{1,n}$$

$$V_{11} = \langle 1 | \hat{V} | 1 \rangle = -\mu_B B_z \langle 200 | \hat{L}_z | 200 \rangle$$

$$\left[\; \underbrace{\hat{H}, \hat{\vec{L}}^2, \hat{L}_z}_{|n l m_e\rangle} \right. - \text{PSKO za Vodonik}$$

$$\left. \hat{L}_z | n l m_e \rangle = m_e \hbar \, | n l m_e \rangle \; \right]$$

$$\hat{L}_z | 200 \rangle = 0 \, | 200 \rangle = 0 \;\Rightarrow\; V_{11} = 0$$

$$V_{12} = \langle 1 | \hat{V} | 2 \rangle = -\mu_B B_z \langle 200 | \hat{L}_z | 210 \rangle = 0$$

$$V_{13} = \langle 1 | \hat{V} | 3 \rangle = -\mu_B B_z \langle 200 | \hat{L}_z | 211 \rangle = 0$$

$$V_{14} = \langle 1 | \hat{V} | 4 \rangle = -\mu_B B_z \langle 200 | \hat{L}_z | 21{-1} \rangle = 0$$

$$V_{21} = \langle 2 | \hat{V} | 1 \rangle = 0$$

$$V_{22} = \langle 2 | \hat{V} | 2 \rangle = 0$$

$$V_{23} = \langle 2 | \hat{V} | 3 \rangle = 0$$

$$V_{24} = \langle 2 | \hat{V} | 4 \rangle = 0$$

$V_{31} = \langle 3|\hat{V}|1\rangle = 0$

$V_{32} = \langle 3|\hat{V}|2\rangle = 0$

$V_{33} = \langle 3|\hat{V}|3\rangle = -\mu_B B_z \hbar$

$V_{34} = \langle 3|\hat{V}|4\rangle = 0$

$V_{41} = \langle 4|\hat{V}|1\rangle = 0$

$V_{42} = \langle 4|\hat{V}|2\rangle = 0$

$V_{43} = \langle 4|\hat{V}|3\rangle = 0$

$V_{44} = \langle 4|\hat{V}|4\rangle = \mu_B B_z \hbar$

$$\begin{vmatrix} -E^{(1)} & 0 & 0 & 0 \\ 0 & -E^{(1)} & 0 & 0 \\ 0 & 0 & -\mu_B B_z \hbar - E^{(1)} & 0 \\ 0 & 0 & 0 & \mu_B B_z \hbar - E^{(1)} \end{vmatrix} = 0$$

Resenja determinante su:

$E_1^{(1)} = 0$ $\qquad$ $E_4^{(1)} = \mu_B B_z \hbar$ $\qquad$ degeneracija je delimično uklonjena

$E_2^{(1)} = 0$

$E_3^{(1)} = -\mu_B B_z \hbar$

$E_{200} = E_2^{(0)} + 0$

$E_{210} = E_2^{(0)} + 0$

$E_{211} = E_2^{(0)} + \mu_B B_z \hbar$

$E_{21-1} = E_2^{(0)} - \mu_B B_z \hbar$

Izračunati prvu popravku energije osnovnog stanja vodoniku-sličnog jona usled konačnih dimenzija jezgra. Samo jezgro smatrati homogenom loptom poluprečnika R i naelektrisanja Ze ravnomerno raspoređenog po zapremini.

Sfera : vlastiena ED

Naelektrisanje sfere $Q = \frac{4\pi}{3} R^3 \rho$

Gausova teorema

$$\oint \vec{E} \cdot d\vec{s} = \frac{\sum q_i}{\varepsilon_0}$$

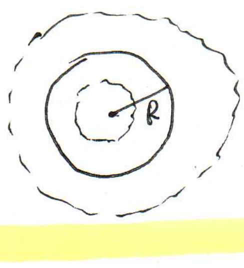
Problem je sferno-simetričan

$r > R \qquad E\,(4\pi r^2) = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$

$r < R \qquad E(4\pi r^2) = \frac{1}{\varepsilon_0} \left(\frac{4\pi}{3} r^3 \rho \right) \Rightarrow$

$$E = \frac{\rho}{3\varepsilon_0} r = \frac{3Q}{4\pi R^3} \frac{r}{3\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0} \frac{r}{R^3}$$

$$E_r = \begin{cases} \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} & r \geq R \\ \frac{Q}{4\pi\varepsilon_0} \frac{r}{R^3} & r < R \end{cases}$$



Potencijalna energija elektrona u polju sfere

$r \geq R$

$$U = -\int_{\infty}^{r} F\,dr = +e\int_{\infty}^{r} E\,dr = \frac{Qe}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{dr}{r^2}$$

$$= \frac{Qe}{4\pi\varepsilon_0}\left.-\frac{1}{r}\right|_{\infty}^{r} = -\frac{Qe}{4\pi\varepsilon_0}\frac{1}{r} \overset{Q=Ze}{=} -\frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{r}$$

$0 < r < R$

$$U = -\int_{\infty}^{R} F\,dr - \int_{R}^{r} F\,dr = +e\int_{\infty}^{R} E\,dr + e\int_{R}^{r} E\,dr$$

$$= \frac{Qe}{4\pi\varepsilon_0}\int_{\infty}^{R}\frac{dr}{r^2} + \frac{Qe}{4\pi\varepsilon_0 R^3}\int_{R}^{r} r\,dr$$

$$= -\frac{Qe}{4\pi\varepsilon_0}\frac{1}{R} + \frac{Qe}{4\pi\varepsilon_0 R^3}\frac{1}{2}\left(r^2 - R^2\right)$$

$$= -\frac{Qe}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{2}\frac{r^2}{R^3} + \frac{1R^2}{2R^3}\right)$$

$$= -\frac{Qe}{4\pi\varepsilon_0}\frac{1}{2R}\left(3 - \frac{r^2}{R^2}\right) \overset{Q=Ze}{=} -\frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{2R}\left(3 - \frac{r^2}{R^2}\right)$$

Dakle, potencijal je

$$U = \begin{cases} -\dfrac{Ze^2}{4\pi\varepsilon_0}\dfrac{1}{r}, & r \geq R \\ -\dfrac{Ze^2}{4\pi\varepsilon_0}\dfrac{1}{2R}\left(3 - \dfrac{r^2}{R^2}\right), & 0 < r < R \end{cases}$$

Примедба: $\int_{\infty}^{r}$ су валидне границе јер је у питању систем са коначном количином наелектрисања!

Potencijal elektrona u polju tačkastog jezgra je $-\frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{r}$ pa je perturbacija

$$\hat{V} = \begin{cases} 0, & r \geq R \\ -\frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{2R}\left(3 - \frac{\hat{r}^2}{R^2}\right) - \left(-\frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{\hat{r}}\right), & 0 < r < R \end{cases}$$

(Јасно, јер је поље ван сфере као да потиче од наелектрисања скоцентрисаног у центру)

Osnovno stanje vodoniku sličnog jona je

$$\psi_{100}(\vec{r}) \equiv R_{10}(r)\, Y_0^0(\theta,\varphi) = 2\left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{-\frac{Zr}{a_0}} \cdot \frac{1}{\sqrt{4\pi}} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-\frac{Zr}{a_0}}$$

$$E^{(1)} = \int_0^R \psi_{100}^*(\vec{r})\, V\, \psi_{100}(\vec{r})\, d^3\vec{r}$$

$R \longrightarrow$ Поправка је због коначних димензија језгра

Budući da je $R \ll a_0$, a obzirom na granice integracije, $\frac{R}{a_0} \to 0 \Rightarrow e^{-\frac{Zr}{a_0}} \to 1$

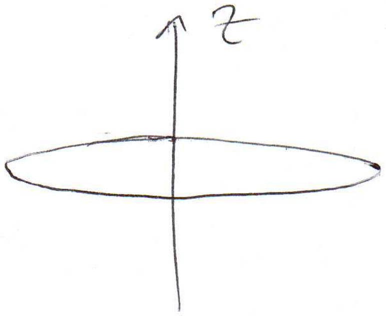
pa je

$$E^{(1)} = \frac{1}{\pi}\left(\frac{Z}{a_0}\right)^3 \int_0^R \left[\frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{r} - \frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{2R}\left(3 - \frac{r^2}{R^2}\right)\right] 4\pi r^2 dr$$

$$= 4\left(\frac{Z}{a_0}\right)^3 \frac{Ze^2}{4\pi\varepsilon_0}\int_0^R \left[\frac{1}{r} - \frac{1}{2R}\left(3 - \frac{r^2}{R^2}\right)\right] r^2 dr$$

$$= 4\left(\frac{Z}{a_0}\right)^3 \frac{Ze^2}{4\pi\varepsilon_0}\frac{R^2}{10}$$

11 Kruti ravanski rotator momenta inercije I i električnog dipolnog momenta $\vec{d}$ se nalazi u slabom spoljašnjem električnom polju. Smatrajući polje za perturbaciju, odrediti prve korekcije energijskih nivoa rotatora.



$$\hat{H} = \frac{\hat{L}_z^2}{2I}$$

$$\hat{L}_z = -i\hbar \frac{\partial}{\partial \varphi} \Rightarrow \hat{L}_z^2 = -\hbar^2 \frac{\partial^2}{\partial \varphi^2}$$

Vremenski nezavisna Šredingerova j-na za rotator

$\hat{H} |\psi\rangle = E|\psi\rangle$, odnosno u koordinatnoj ~~repr~~ reprezentaciji

$$-\frac{\hbar^2}{2I} \frac{\partial^2 \psi}{\partial \varphi^2} = E\psi(\varphi) \qquad (*)$$

$$[\hat{H}, \hat{L}_z] = 0 \Rightarrow E_m = \frac{m^2\hbar^2}{2I}$$

Sada $(*) \Rightarrow$

$$\psi'' = -\frac{2I}{\hbar^2} E_m \psi = -m^2 \psi$$

$$\psi'' + m^2 \psi = 0$$

Opste resenje

$$\psi = c_1 e^{im\varphi} + c_2 e^{-im\varphi} \qquad (**)$$

Komutiranje sa $\hat{L}_z$ ograničava oblik rešenja
($(**)$ nije svojstveno stanje za $\hat{L}_z$)

$$\psi = c_1 e^{im\varphi}$$

Normiranje

$$\int_0^{2\pi} |\psi(\varphi)|^2 d\varphi = 1 \Rightarrow c_1 = \frac{1}{\sqrt{2\pi}}$$

pa je $\psi(\varphi) = \frac{1}{\sqrt{2\pi}} e^{im\varphi}$

Uslov periodičnosti

$$\psi(0) = \psi(2\pi) \Rightarrow e^{2\pi m i} = 1 \Rightarrow$$

$m = 0, \pm 1, \pm 2, \pm 3$

pa i $E_m = \frac{\hbar^2 m^2}{2I}$, $m = 0, \pm 1, \pm 2, \pm 3 \ldots$

E_m-ovi su degenerisani za $m \neq 0$

Prvo popravka za $m=0$

$$E^{(1)} = \langle \psi_0 | \hat{V} | \psi_0 \rangle$$

$$\hat{V} = -\hat{\vec{d}} \cdot \vec{E} = -E d \cos\varphi$$

$$E^{(1)} = -Ed\int_0^{2\pi} \psi_0^*(\varphi)\cos\varphi\, \psi_0(\varphi)\, d\varphi$$

$$= -\frac{Ed}{2\pi}\int_0^{2\pi}\cos\varphi\, d\varphi = 0$$

Za $m \neq 0$, svojstvene vrednosti za energiju su degenerisane, pa treba izračunati popravke putem sekularne jednačine.

Neka je:

$$\psi_m(\varphi) = \frac{1}{\sqrt{2\pi}} e^{im\varphi} \equiv |1\rangle$$ (имe y вид у нет. вред., тј. $m<0$)

$$\psi_n(\varphi) = \frac{1}{\sqrt{2\pi}} e^{in\varphi} \equiv |2\rangle$$ ($n>0$)

$$\begin{vmatrix} V_{11} - E^{(1)} & V_{12} \\ V_{21} & V_{22} - E^{(1)} \end{vmatrix} = 0$$

$$V_{11} = \langle 1|\hat{V}|1\rangle = -\frac{Ed}{2\pi}\int_0^{2\pi}\cos\varphi\, d\varphi = 0$$

Slično i

$$V_{22} = \langle 2|\hat{V}|2\rangle = 0$$

$$V_{12} = \langle 1|\hat{V}|2\rangle = -\frac{Ed}{2\pi}\int_0^{2\pi} e^{i(m-n)\varphi}\cos\varphi\, d\varphi$$

Integral

$$I = \frac{-i\left(-1+e^{2i(m-n)\pi}\right)(m-n)}{-1+(m-n)^2} \qquad (*)$$

$$V_{12} = -\frac{Ed}{2\pi} I$$

Jasno je da je $V_{21} = -\frac{Ed}{2\pi} I^*$

Onda sekularna j-na daje

$$\left(E^{(1)}\right)^2 - \left(\frac{Ed}{2\pi}\right)^2 I I^* = 0$$

$$\left(E^{(1)}\right)^2 = \left(\frac{Ed}{2\pi}\right)^2 |I|^2 \Rightarrow$$

$$E^{(1)} = \pm \frac{Ed}{2\pi}|I|$$

$$|I| = \sqrt{I I^*} = \dots = \sqrt{2}\sqrt{1-\cos 2(m-n)\pi}\ \left|\frac{m-n}{(m-n)^2-1}\right|$$

Obzirom na to da je $m=-n$, iz posledenjeg izraza sledi da je $|I|=0$, tj $E^{(1)}$ su nula za svaku vrednost kvantnog broja m.

2. Na česticu mase m koja se nalazi u beskonačno-dubokoj dvodimenzionalnoj potencijalnoj jami $(0<x<L, 0<y<L)$ deluje perturbacija $V(x,y)=V_0\cos^2\frac{\pi x}{L}$. Odrediti razdvajanje trećeg pobuđenog nivoa u prvom redu računa perturbacije.

Sv. f-je neperturbovanog hamiltonijana su

$$\psi^{(0)}_{n,k}(x,y)=\frac{2}{L}\sin\frac{n\pi x}{L}\sin\frac{k\pi y}{L}, \quad n,k=1,2,\ldots$$

a odgovarajuće svojstvene energije

$$E^{(0)}_{nk}=\frac{\hbar^2\pi^2}{2mL^2}(n^2+k^2)$$

za $n=1, k=2$ i za $n=2, k=1$ energija je

$$E^{(0)}_3=\frac{5\hbar^2\pi^2}{2mL^2}$$

a stanja koja odgovaraju ovoj energiji

$$\psi^{(0)}_{1,2}(x,y)=\frac{2}{L}\sin\frac{\pi x}{L}\sin\frac{2\pi y}{L} \quad i$$

$$\psi^{(0)}_{2,1}(x,y)=\frac{2}{L}\sin\frac{2\pi x}{L}\sin\frac{\pi y}{L}$$

Neka je, kraće, $\psi^{(0)}_{1,2}(x,y)=|1\rangle$ a $\psi^{(0)}_{2,1}(x,y)=|2\rangle$

Sekularna jednačina

$$\begin{vmatrix} V_{11}-E^{(1)} & V_{12} \\ V_{21} & V_{22}-E^{(1)} \end{vmatrix} = 0$$

$$V_{11} = \langle 1 | \hat{V} | 1 \rangle = \frac{4}{L^2} \int_0^L \int_0^L dx\,dy\, V_0 \cos^2\frac{\pi x}{L} \sin^2\frac{\pi x}{L} \sin^2\frac{2\pi y}{L}$$

$$= \frac{4V_0}{L^2} \int_0^L dx \cos^2\frac{\pi x}{L} \sin^2\frac{\pi x}{L} \int_0^L \sin^2\frac{2\pi y}{L}\, dy$$

[Trigonometrijski identiteti

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha \qquad (1)$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \qquad (2)$$

Iz (1) $\Rightarrow$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$\sin\frac{\pi x}{L}\cos\frac{\pi x}{L} = \frac{1}{2}\sin\frac{2\pi x}{L} \Rightarrow$$

$$\sin^2\frac{\pi x}{L}\cos^2\frac{\pi x}{L} = \frac{1}{4}\sin^2\frac{2\pi x}{L}$$]

$$= \frac{V_0}{L^2} \int_0^L dx \sin^2\frac{2\pi x}{L} \int_0^L dy \sin^2\frac{2\pi y}{L}$$

[$\sin^2\xi = \frac{1}{2}(1-\cos 2\xi)$]

$$I = \int_0^L \sin^2 \frac{2\pi x}{L}\, dx = \frac{1}{2}\int_0^L \left(1 - \cos\frac{4\pi x}{L}\right) dx$$

$$= \frac{1}{2}\left(\int_0^L dx - \int_0^L \cos\frac{4\pi x}{L}\, dx \right)$$

$$= \frac{1}{2}\left(L - \frac{L}{4\pi} \underbrace{\sin\frac{4\pi x}{L}}_{0} \Big|_0^L \right)$$

$$= \frac{1}{2} L$$

a isto i za $\int_0^L \sin^2 \frac{2\pi y}{L}\, dy = \frac{1}{2} L$

$$V_{11} = \frac{V_0}{L^2}\frac{L^2}{4} = \frac{V_0}{4}$$ (ТРЕБАЛО БИ ДА ЈЕ V_0)

$$V_{12} = \langle 1|\hat{V}|2\rangle = \frac{4}{L^2}\int_0^L\int_0^L dx\,dy\, V_0 \cos^2\frac{\pi x}{L} \sin\frac{2\pi x}{L} \sin\frac{\pi x}{L}$$

$$\sin\frac{2\pi x}{L} \sin\frac{\pi y}{L} = 0$$ (koristiti identitete)

a samim tim i $V_{21} = 0$

$$V_{22} = \frac{V_0}{2}$$ (Domaći)

$$\begin{vmatrix} \frac{V_0}{4} - E^{(1)} & 0 \\ 0 & \frac{V_0}{2} - E^{(1)} \end{vmatrix} = 0$$

~~Nule~~ polinoma su $\frac{V_0}{2}$ i $\frac{V_0}{4}$

Svojstveni nivoi perturbovanog Hamiltonijana

$$E_3' = E_3^{(0)} + E_1^{(1)} = \frac{5\hbar^2\pi^2}{2mL^2} + \frac{V_0}{2}$$

$$E_3'' = E_3^{(0)} + E_2^{(1)} = \frac{5\hbar^2\pi^2}{2mL^2} + \frac{V_0}{4}$$

odnosno $E_3^{(0)}$ se cepa na E_3' i E_3''!

Više nema degeneracije datog nivoa.

Fraza: Perturbacija uklida degeneraciju svojstvene vrednosti (energijskog nivoa)